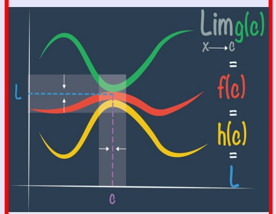
Math 261

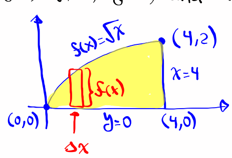
Fall 2022

Lecture 45

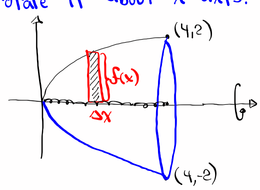


Feb 19-8:47 AM

1) Draw the region enclosed by $f(x) = \sqrt{x}$, $y=0$, and $x=4$.



Now let's take the region and rotate it about x-axis.



2) find its area

$$A = \int_0^4 f(x)\,dx$$

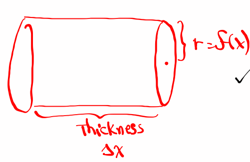
$$= \int_0^4 \sqrt{x}\,dx$$

$$= \frac{x^{3/2}}{3/2}\Big|_0^4$$

$$= \frac{2}{3} x\sqrt{x}\Big|_0^4$$

$$= \frac{2}{3}\cdot 4\sqrt{4} - 0$$

$$= \boxed{\frac{16}{3}}$$



✓ Region is 100% attached to the axis of Rev.

✓ Ref. Rectangle is ⊥ to axis of Revolution.

Use Disk Method

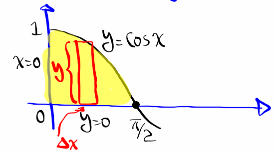
$$V = \int_a^b \pi\,[f(x)]^2\,dx$$

$$V = \int_0^4 \pi\left[\sqrt{x}\right]^2 dx = \pi\int_0^4 x\,dx = \pi\cdot\frac{x^2}{2}\Big|_0^4 = \boxed{8\pi}$$

Nov 17-8:48 AM

Consider the enclosed region by $x=0$, $y=0$, and $y=\cos x$ in QI.

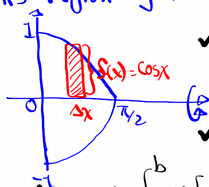
1) Draw the region.



2) Find its area.

$$A=\int_0^{\pi/2}\cos x\,dx=\sin x\Big|_0^{\pi/2}=\sin\tfrac{\pi}{2}-\sin 0=\boxed{1}$$

3) Rotate this region by x-axis, and find its volume.



✓ 1) Region 100% attached to A.O.R.

✓ 2) Ref. Rec. ⊥ A.O.R.

Disk Method $V=\int_a^b \pi\left[f(x)\right]^2dx$

See notes from Yesterday $\int \cos^2x\,dx=\frac{1}{2}x+\frac{1}{4}\sin 2x+C$

$$V=\int_0^{\pi/2}\pi\left[\cos x\right]^2dx=\pi\int_0^{\pi/2}\cos^2x\,dx$$

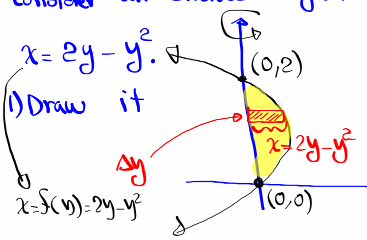
$$=\pi\left[\frac{1}{2}x+\frac{1}{4}\sin 2x\right]\Big|_0^{\pi/2}=\pi\left[\frac{1}{2}\cdot\frac{\pi}{2}+\cancel{\frac{1}{4}\sin\pi}^{\,0}\right]-\pi\left[0+\cancel{\frac{1}{4}\sin 0}^{\,0}\right]$$

$$=\boxed{\frac{\pi^2}{4}}$$

Nov 17-9:02 AM

Consider an enclosed region by $x=0$ and $x=2y-y^2$.

1) Draw it



2) Find its area

$$A=\int_0^2(2y-y^2)\,dy=\left(y^2-\frac{y^3}{3}\right)\Big|_0^2=4-\frac{8}{3}=\boxed{\frac{4}{3}}$$

3) Find the volume if the region is rotated about y-axis.

Disk
1) Region 100% attached to A.O.R.
2) Ref. Rec. ⊥ A.O.R.

$$V=\int_c^d\pi\left[f(y)\right]^2dy$$

$$=\int_0^2\pi\left[2y-y^2\right]^2dy=\pi\int_0^2(4y^2-4y^3+y^4)\,dy$$

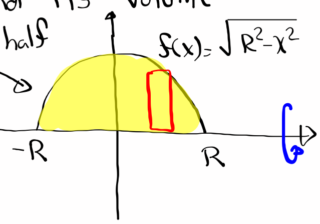
$$=\pi\left[\frac{4y^3}{3}-y^4+\frac{y^5}{5}\right]\Big|_0^2$$

$$=\pi\left[\frac{32}{3}-16+\frac{32}{5}\right]=\boxed{\frac{16\pi}{15}}$$

Nov 17-9:13 AM

Rotate the region given below by x-axis, and find its volume

Top half of a Circle



1) is region 100% attached to A.O.R.? Yes

2) Is Ref. Rec. $\perp$ A.O.R.? Yes

$\to$ Disk

$$V = \int_a^b \pi \left[f(x) \right]^2 dx$$

$$V = \int_{-R}^{R} \pi \left[\sqrt{R^2 - x^2} \right]^2 dx$$

$$= \pi \int_{-R}^{R} (R^2 - x^2)\, dx$$

$$= 2\pi \int_{0}^{R} (R^2 - x^2)\, dx$$

$$= 2\pi \left[R^2 x - \frac{x^3}{3} \right] \Bigg|_0^R = 2\pi \left[R^3 - \frac{R^3}{3} \right] = 2\pi \cdot \frac{2R^3}{3} = \boxed{\frac{4\pi R^3}{3}}$$

Nov 17-9:27 AM